

18. Using the Laplace transform, Solve

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 5y = e^{-x} \sin \sin x \text{ where } y(0) = 0 \quad \text{and} \\ y(0) = 1.$$

19. Construct the character table for C_{2v} and C_{3v} .

20. State and prove Laplace — de Moivre limit theorem of probability.

GPH21/

DPH21 — MATHEMATICAL PHYSICS – II

APRIL/MAY 2023

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Check whether the following functions are analytic.

(a) $|Z|$

(b) $\operatorname{Re} Z$

2. Mention any two properties of complex line integrals.

3. If $U = 2x^2 + 3xy$, find $\frac{\partial u}{\partial x}$.

4. Write the Laplace equation in Cartesian coordinates.

5. Give the shifting property of Fourier transform.

6. Find the Laplace transform of e^{at} and e^{-at} .

7. Define order of a group.
8. State orthogonality theorem.
9. What is meant by random variable?
10. List out the properties of normal curve.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Determine the analytic function, whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$.

Or

- (b) State and prove Taylor series.
12. (a) Obtain the solution of Laplace equation in spherical polar coordinates.

Or

- (b) Determine the D' Alembert's solution of the vibrating string.
13. (a). State and prove Faltung theorem.

Or

- (b) Find the Fourier sine and cosine transforms of the function

$$F(x) = x^2, 0 < x < 4.$$

14. (a) Distinguish between isomorphism and homomorphism.

Or

- (b) Prove that the group of order three is always cyclic.

15. (a) What is Binomial distribution. derive the constants first four moments, mode and moment generally function of Binomial distribution.

Or

- (b) Define probability. Differentiate between a priori probability a posterior probability.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Apply the calculus of residue to show that

$$\int_0^{2\pi} \frac{\sin^2 \theta}{a + b \cos \theta} d\theta = \frac{2\pi}{b^2} \left(a - \sqrt{a^2 - b^2} \right); a > b > 0.$$

17. Obtain the solution of diffusion equation.